## SPIRALING ALONG THE (N-DIMENSIONAL) CLOCK-TOWER:

a tour of the layered dimensions of music

## Torus (a doughnut)



## Lesson:

- "If I have learned nothing else in the course of this research, I have learned to perceive n-dimensional doughnuts in the structural fibers of music."


## Mind Map

- Dimensions

ㅁ Definitions
Z Z12, cycles, and music as numbers

- Mind-expanding Pictures
- "Giant Steps" / Coltrane Matrix
- Taken it Further: Topological Spaces


## Welcome to the Tower

I I am the 3-D alien to your Flatland.

## Dimensional Analogy

- A dimensional analogy is a tool for visualizing n-dimensions, by a projection onto a (n-1) system.
- 4-D Soup
- Dimension- the minimum number of coordinates needed to specify all the points in a system.


## Definitions/Notation

- Pitch- in equal tempered tuning, each octave is divided into 12 even pitches called semitones.

ㅁ S- $\{0,1,2,3,4,5,6,7,8,9,10,11\}$

- P-\{C,C\#, D,D\#, E, F,F\#, G,G\#, A, A\#, B \} C' ...


## Definitions/Notation

- Interval- the distance between any two pitches, which we will measure in number of semitones.
- Octave- the harmonic ratio of 2:1 higher or lower than a given pitch.
Terminology from the 7-note diatonic scale ( $8^{\text {th }}$ note is same as first) western school of thought, which assigns the class of root, second, third, fourth, fifth, sixth, seventh, and octave to the members of a scale.


## Definitions/Notation

- Scale- a collection of pitches which map out a path through the octave.
- Chromatic scale C-\{0,1,2,3,4,5,6,7,8,9,10,11,12/0\}=
■ C-(C, C\#,D,D\#,E,F,F\#,G,G\#,A,A\#,B,C...\}
- Cycle Classes- all cycles are combinations of the following classes and their retrogrades:


## Definitions/Notation

- Cycle classes:

ㅁ $\{0\}-(0,12,24, \ldots) / /$ same as 12
ㅁ $\{1\}-(0,1,2,3,4,5,6,7,8,9,10,11)$

- $\{2\}-(0,2,4,6,10)$
- $\{3\}-(0,3,6,9)$
- $\{4\}-(0,4,8)$

ㅁ $\{5\}-(5,10,3,8,1,6,11,4,9,2,7)$

- $\{6\}-(0,6)$

ㅁ $\{7\}=$ inverse of $\{5\}$

## Z12

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 2 | 2 | 4 | 6 | 8 | 10 | 0 | 2 | 4 | 6 | 8 | 10 |
| 3 | 3 | 6 | 9 | 0 | 3 | 6 | 9 | 0 | 3 | 6 | 9 |
| 4 | 4 | 8 | 0 | 4 | 8 | 0 | 4 | 8 | 0 | 4 | 8 |
| 5 | 5 | 10 | 3 | 8 | 1 | 6 | 11 | 4 | 9 | 2 | 7 |
| 6 | 6 | 0 | 6 | 0 | 6 | 0 | 6 | 0 | 6 | 0 | 6 |
| 7 | 7 | 2 | 9 | 4 | 11 | 6 | 1 | 8 | 3 | 10 | 5 |
| 8 | 8 | 4 | 0 | 8 | 4 | 0 | 8 | 4 | 0 | 8 | 4 |
| 9 | 9 | 6 | 3 | 0 | 9 | 6 | 3 | 0 | 9 | 6 | 3 |
| 10 | 10 | 8 | 6 | 4 | 2 | 0 | 10 | 8 | 6 | 4 | 2 |
| 11 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |

## Z12

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 2 | 2 | 4 | 6 | 8 | 10 | 0 | 2 | 4 | 6 | 8 | 10 |
| 3 | 3 | 6 | 9 | 0 | 3 | 6 | 9 | 0 | 3 | 6 | 9 |
| 4 | 4 | 8 | 0 | 4 | 8 | 0 | 4 | 8 | 0 | 4 | 8 |
| 5 | 5 | 10 | 3 | 8 | 1 | 6 | 11 | 4 | 9 | 2 | 7 |
| 6 | 6 | 0 | 6 | 0 | 6 | 0 | 6 | 0 | 6 | 0 | 6 |
| 7 | 7 | 2 | 9 | 4 | 11 | 6 | 1 | 8 | 3 | 10 | 5 |
| 8 | 8 | 4 | 0 | 8 | 4 | 0 | 8 | 4 | 0 | 8 | 4 |
| 9 | 9 | 6 | 3 | 0 | 9 | 6 | 3 | 0 | 9 | 6 | 3 |
| 10 | 10 | 8 | 6 | 4 | 2 | 0 | 10 | 8 | 6 | 4 | 2 |
| 11 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |

## Z12

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 2 | 2 | 4 | 6 | 8 | 10 | 0 | 2 | 4 | 6 | 8 | 10 |
| 3 | 3 | 6 | 9 | 0 | 3 | 6 | 9 | 0 | 3 | 6 | 9 |
| 4 | 4 | 8 | 0 | 4 | 8 | 0 | 4 | 8 | 0 | 4 | 8 |
| 5 | 5 | 10 | 3 | 8 | 1 | 6 | 11 | 4 | 9 | 2 | 7 |
| 6 | 6 | 0 | 6 | 0 | 6 | 0 | 6 | 0 | 6 | 0 | 6 |
| 7 | 7 | 2 | 9 | 4 | 11 | 6 | 1 | 8 | 3 | 10 | 5 |
| 8 | 8 | 4 | 0 | 8 | 4 | 0 | 8 | 4 | 0 | 8 | 4 |
| 9 | 9 | 6 | 3 | 0 | 9 | 6 | 3 | 0 | 9 | 6 | 3 |
| 10 | 10 | 8 | 6 | 4 | 2 | 0 | 10 | 8 | 6 | 4 | 2 |
| 11 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |

## Z12

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 2 | 2 | 4 | 6 | 8 | 10 | 0 | 2 | 4 | 6 | 8 | 10 |
| 3 | 3 | 6 | 9 | 0 | 3 | 6 | 9 | 0 | 3 | 6 | 9 |
| 4 | 4 | 8 | 0 | 4 | 8 | 0 | 4 | 8 | 0 | 4 | 8 |
| 5 | 5 | 10 | 3 | 8 | 1 | 6 | 11 | 4 | 9 | 2 | 7 |
| 6 | 6 | 0 | 6 | 0 | 6 | 0 | 6 | 0 | 6 | 0 | 6 |
| 7 | 7 | 2 | 9 | 4 | 11 | 6 | 1 | 8 | 3 | 10 | 5 |
| 8 | 8 | 4 | 0 | 8 | 4 | 0 | 8 | 4 | 0 | 8 | 4 |
| 9 | 9 | 6 | 3 | 0 | 9 | 6 | 3 | 0 | 9 | 6 | 3 |
| 10 | 10 | 8 | 6 | 4 | 2 | 0 | 10 | 8 | 6 | 4 | 2 |
| 11. | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |

## Circle of Fifths



Figure 2: Cycle of Fifths (P4/P5)

## Cycle Diagrams



Figure 1: Chromatic Cycle (m2/M7)

## Cycle Diagrams



## Coltrane Matrix/ "Giant Steps"

$\begin{array}{lllll}0 & 3 & 811470\end{array}$
47038114
710361127
111794511

## Coltrane Matrix/ "Giant Steps"

0-----8-----4-----0
$0 \quad 3811470$
47038114
710361127
111794511

## Coltrane Matrix

- The Coltrane Matrix is an expansion on the augmented cycle, with each chord preceded by the a chord a perfect fifth above the desired chord.


## Transformations

- Octave- which octave a pitch is in
- Transposition- moves every member of musical object by an equal amount.
- Permutation- changes the order of the elements

Inversion- turns the system upside down

- Cardinality- inserts duplicates into the system


## Transformations

- Due to this expansion of the discrete system of numbers into Real spaces, new geometrical musical systems, based in topology, are able to handle continuous music functions and other forms of tuning.


## Pulling the pieces together

ㅁ. Now that we are aware of circular pitch diagrams, modular cycles, numeric representation of musical objects, and we understand the fact that more than three coordinates are required to describe a group of musical objects, we are ready for the following idea.

## Tower

$\square$ Pitch class is along the ring of pitches, octave is the height of the ring, and n-is the number of independent voices or instruments, and all of this interacts in time.

- Image stacks of rotating rings sliding along a dimension of time.


## Summary

- Listening to music can be a trans-dimensional experience.


