

# Knapsack Problems

MATH 3220

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# Outline

- Introduction
- Components
- Why the Knapsack Problem?
- Techniques
- Solving KP
- Variations

# Introduction

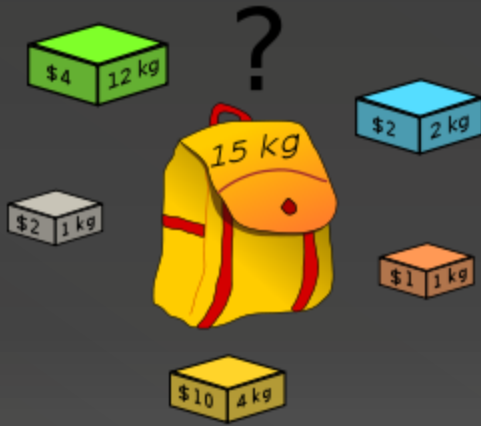
Congratulations! You just won a trip to Hawaii!!

- Your Items:  
Different Values and  
Different Costs
- How will you decide  
what to pack?



(Jennifer Huls Photography)

# Components



(Wikipedia.com)

- Originates : Must fill knapsack with items that are most valuable
- Studied since 1897
- Mathematician Tobias Dantzig (1884-1956)
- Folklore

# Components

## Defining the Problem

### **The 0-1 Knapsack Problem:**

- Need unique objects (one time use)
  - Value for each object
  - Cost for each object
- Problem Objective: Highest value within cost limit
- Example: Your house is on fire and must save your pets!



(Carol Hawkins Studios)

# Why the Knapsack Problem?

- ◎ **Useful to Real Life Problem Solving**
  - > Determining least wasteful ways to cut raw materials
  - > Selection of capital investments
  - > Creation and scoring of tests

# Techniques

- Branch and Bound
- Dynamic Programming
- Hybridizations of both

# Solving KP

- ◎ Using Bounded Knapsack Problem:
  - > You were given one suitcase
  - > The capacity of this suitcase is 50 pounds
  - > You have to choose from the items listed to pack
  - > You may pack more than one item only if that item is listed twice



# Solving KP

| Item       | Value | Cost (Weight) |
|------------|-------|---------------|
| Swim Suit  | 400   | 5 lbs.        |
| Flip Flops | 200   | 10 lbs.       |
| Hair Dryer | 350   | 20 lbs.       |
| Camera     | 500   | 30 lbs.       |
| Swim Suit  | 400   | 5 lbs.        |
| Shorts     | 200   | 5 lbs.        |
| High Heels | 150   | 10 lbs.       |
| Snorkel    | 150   | 5 lbs.        |
| Sunscreen  | 500   | 20 lbs.       |
| Hat        | 300   | 10 lbs.       |

The weights listed above are not accurate.

# Solving KP

How do we choose what to pack?

| Item       | Value | Cost (Weight) |
|------------|-------|---------------|
| Swim Suit  | 400   | 5 lbs.        |
| Flip Flops | 200   | 10 lbs.       |
| Hair Dryer | 350   | 20 lbs.       |
| Camera     | 500   | 30 lbs.       |
| Swim Suit  | 400   | 5 lbs.        |
| Shorts     | 200   | 5 lbs.        |
| High Heels | 150   | 10 lbs.       |
| Snorkel    | 150   | 5 lbs.        |
| Sunscreen  | 500   | 20 lbs.       |
| Hat        | 300   | 10 lbs.       |

## Greedy Solution

-Choose the highest values within capacity first

-Results:

Cost: 50 lbs.

Value: 1000

# Solving KP

- How do we choose what to pack?

| Item       | Value | Cost (Weight) |
|------------|-------|---------------|
| Swim Suit  | 400   | 5 lbs.        |
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| Hat        | 300   | 10 lbs.       |

## Optimal Solution

-Results:

**Cost: 50 lbs.**

**Value: 1950**

# Variations

- 0-1 Knapsack Problem
- Bounded Knapsack Problem
- Unbounded Knapsack Problem
- Subset-sum Problem

# Variations

- ⦿ Change-making Problem
- ⦿ 0-1 Multiple Knapsack Problem
- ⦿ Generalized Assignment Problem
- ⦿ Bin-packing Problem

# Review

- Introduction
- Components
- Why the Knapsack Problem?
- Techniques
- Solving KP
- Variations

# References

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# Questions?



(Six Mistakes When Asking Questions)