KNAPSACK PROBLEMS: What Can A Beautiful Dancer Fit In Her Dance Bag?

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Overview:
Can I Fit It All In?

- What is a Knapsack Problem?

- Basic Mathematic Form

- Types of Knapsack Problems

- How are they solved?

- Examples

- Why do we care about KP?

What is a Knapsack Problem?

- Suppose a beautiful dancer, let's just say Tara for our purposes, needs to fill her dance bag for the day.

- She can choose from various items that will maximize her comfort for the day.

- How does she do this?

Mathematically Speaking…
In Its Most Basic Form:

Maximize:

$$\sum_{j=1}^{n} p_j x_j$$

Subject to:

$$\sum_{j=1}^{n} w_j x_j \leq W,$$

P_j = profit

W_j = weight

W = max weight that can be in bag

$$x_j \in \{0, 1\}$$

for all: $\quad 1 \leq j \leq n$

Types of Knapsack Problems:

- 0-1 KP
- Bounded KP
- Unbounded KP
- Subset-Sum Problem
- Multiple Choice KP
- Multiple KP

How Is It Solved?
This Dancer is Confused.

- KP is NP-Complete.
- KP can be solved in pseudo-polynomial time using dynamic programming.
- Martello and Toth proposed a Greedy Approximation Algorithm to solve KP

Dynamic Programming Solution

- Consists of considering i stages and computing at each stage $n > 1$ recursive values given costs $c_1 \ldots c_n$ and corresponding values $v_1 \ldots v_n$.

- We want to maximize the total value subject to the constraint that total cost < C.

- For each $i \leq C$, define $A(i)$ to be the maximum value that can be attained with total cost less than or equal to i.

- Tabulating the results $A(0)$ up through $A(C)$ gives the solution.

Greedy Approximation Algorithm

- Sorts the essentials in decreasing order
- Inserts them into the sack starting with the first and greatest element
- Inserts until there is no longer space for more
- If k is the maximum number of essentials that can fit, the algorithm is guaranteed to insert at least k/2 of them

Why Do We Care?

- KP have been intensively studied in the last decade attracting theorists and practicians

- An impressive amount of research has been published

- These problems can model many industrial situations:
  - Capital Budgeting
  - Cargo Loading
  - Cutting Stock

Why Do We Care? Cont'd

- Subset sum has been used in cryptography.

- However, Merkle-Hellman and several similar algorithms created to solve subset sum were broken because they were solvable in polynomial time.

- KP is also used in combinatorics, applied mathematics, and complexity theory.

The Beautiful Dancer is Happy!

- She is dancing with happiness because she can get the most out of the day by what is in her dance bag!

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