COMPUTATIONAL EXAMPLE OF THE SIMPLE ANALYSIS OF VARIANCE (DREAM EXAMPLE)

A. Data	Group 1 (No Deprivation)	Group 2 (Some Deprivation)	Group 3 (Much Deprivation)	Total Sample
	7	5	21	
	5	9	15	
	3	12	17	
	4	12	18	
	1	7		
		3		
Totals	$T_1 = (7 + 5 + \cdots + 1)$	$T_2 = (5 + 9 + \cdots + 3)$	$T_3 = (21 + 15 + \cdots + 18)$	$T = \sum_{j=1}^{p} T_j$
	$T_1 = 20$	$T_2 = 48$	$T_3 = 71$	$= 139$
n_j	$n_1 = 5$	$n_2 = 6$	$n_3 = 4$	$N = \sum_{j=1}^{p} n_j = 15$
Means	$\bar{X}_1 = \frac{20}{5} = 4.00$	$\bar{X}_2 = \frac{48}{6} = 8.00$	$\bar{X}_3 = \frac{71}{4} = 17.75$	
Sum of squared scores	$\Sigma X_{i1}^2 = 100$	$\Sigma X_{i2}^2 = 452$	$\Sigma X_{i3}^2 = 1279$	$\sum_{j=1}^{p}\left(\sum_{i=1}^{n_j} X_{ij}^2\right) = 1831$
Squared sum of scores divided by n_j	$\frac{T_1^2}{n_1} = \frac{(20)^2}{5} = 80$	$\frac{T_2^2}{n_2} = \frac{(48)^2}{6} = 384$	$\frac{T_3^2}{n_3} = \frac{(71)^2}{4} = 1260.25$	$\sum_{j=1}^{p}\left(\frac{T_j^2}{n_j}\right) = 1724.25$

B. $(\text{I}) = \frac{T^2}{N} = \frac{(139)^2}{15} = 1288.07 \qquad (\text{II}) = \sum_{j=1}^{p}\left(\sum_{i=1}^{n_j} X_{ij}^2\right) = 1831 \qquad (\text{III}) = \sum_{j=1}^{p}\left(\frac{T_j^2}{n_j}\right) = 1724.25$

C. $SS_{between} = (\text{III}) - (\text{I}) = 1724.25 - 1288.07 = 436.18 \qquad df = c - 1 = 3 - 1 = 2$

$SS_{within} = (\text{II}) - (\text{III}) = 1831 - 1724.25 = 106.75 \qquad df = N - c = 15 - 3 = 12$

$SS_{total} = (\text{II}) - (\text{I}) = 1831 - 1288.07 = 542.93 \qquad df = N - 1 = 15 - 1 = 14$

$MS_{between} = \frac{SS_{between}}{df_{between}} = \frac{436.18}{2} = 218.09 \qquad MS_{within} = \frac{SS_{within}}{df_{within}} = \frac{106.75}{12} = 8.90$

D. Summary Table

Source	df	SS	MS	F
Between groups	2	436.18	218.09	$\frac{MS_{between}}{MS_{within}} = \frac{218.09}{8.90} = 24.50^{**}$
Within groups	12	106.75	8.90	
Total	14	542.93		

Critical values ($df = 2, 12$) $^{*}F_{.05} = 3.88,\ p < .05$

$^{**}F_{.01} = 6.93,\ p < .01$

THE ANALYSIS-OF-VARIANCE TABLE

Since several steps are involved in the computation of both the between- and within-group variances, the entire set of results may be organized into an *analysis-of-variance (ANOVA) table*.

Analysis-of-variance table

Source	Sum of squares	Degrees of freedom	Mean square (variance)	F
Between	$SSB = \sum_j n_j(\bar{X}_j - \bar{\bar{X}})^2$	$c-1$	$MSB = SSB/(c-1)$	MSB/MSW
Within	$SSW = \sum_i \sum_j (X_{ij} - \bar{X}_j)^2$	$n-c$	$MSW = SSW/(n-c)$	
Total	$SST = \sum_i \sum_j (X_{ij} - \bar{\bar{X}})^2$	$n-1$		

Analysis-of-variance table using computational formulas

Source	Sum of squares	Degrees of freedom	Mean square	F
Between	$SSB = \sum_j \frac{T_j^2}{n_j} - \frac{(GT)^2}{n}$	$c-1$	$\frac{SSB}{c-1}$	$\frac{MSB}{MSW}$
Within	$SSW = \sum_i \sum_j X_{ij}^2 - \sum_j \frac{T_j^2}{n_j}$	$n-c$	$\frac{MSW}{n-c}$	
Total	$SST = \sum_i \sum_j X_{ij}^2 - \frac{(GT)^2}{n}$	$n-1$		

A consumer organization wanted to compare the price of a particular toy in three types of Stores in a suburban county: discount toy stores, department stores, and variety stores. A random sample of 5 discount toy stores, 6 department stores, and 5 variety stores was selected. The results were as follows:

Discount Toy	Department	Variety
12	15	22
14	18	27
15	14	23
16	18	25
16	17	25
	22	

Is there evidence of a difference in average price among the types of stores at $\alpha = 0.01$?